

## Two-Stage Sampling for Age Distribution in the Atlantic Menhaden Fishery, with Comments on Optimal Survey Design

ALEXANDER J. CHESTER AND JAMES R. WATERS

National Marine Fisheries Service  
Southeast Fisheries Center  
Beaufort Laboratory  
Beaufort, North Carolina 28516-9722

### ABSTRACT

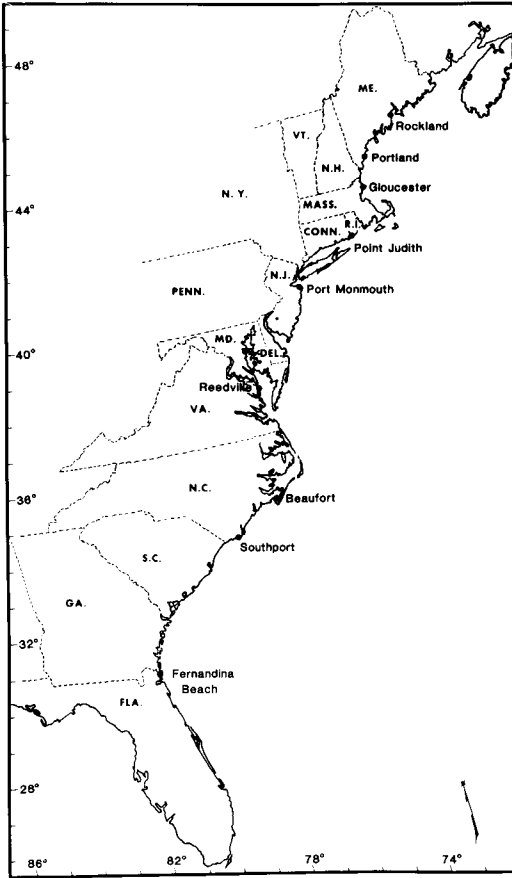
Principles of probability sampling theory and optimization of survey design have been applied to the estimation of age composition in the Atlantic menhaden (*Brevoortia tyrannus*) purse-seine fishery. These methods provide a framework within which stock assessment scientists can determine confidence intervals about estimated parameters and can increase the overall efficiency of sampling programs. The Atlantic menhaden survey is discussed with respect to required assumptions and serves to illustrate several problems commonly encountered in imposing theoretical requirements on the sampling of commercial catches. A two-stage design is developed in which purse-seine sets and fish within sets are randomly sampled. Formulas are given for estimating mean age composition with joint 95% confidence intervals for a single week and for the entire fishing season. In addition, we recommend a procedure for determining optimal survey design which admits a class of potential solutions that has been previously overlooked when several parameters (e.g., age proportions) are measured simultaneously.

The confidence that fishery scientists and managers place in stock assessment conclusions is limited by the accuracy and precision of routinely collected fishery statistics. Several recent publications have applied theories of probability sampling and economic optimization to assess the overall reliability of fishery statistics and determine efficient survey designs (Kutkuhn 1963, for salmon; Tomlinson 1971, for northern anchovy; Southward 1976, for halibut; Brennan and Palmer 1978, for mackerel; Schweigert and Sibert 1983, for Pacific herring). A survey design commonly recommended for purse-seine fisheries has been two-stage cluster sampling (Pope 1956; Tomlinson 1971; Collins 1971; Schweigert and Sibert 1983). In this design, the first stage consists of randomly sampling  $n$  primary units (usually boatloads or purse-seine sets), and the second stage consists of randomly sampling  $m$  elements (fish) from each selected set. Thus, estimates are obtained for the among set and within set components of variance. The relative size of these variance components and the relative costs of sampling sets and individual fish determine the optimal number of sets and fish to be sampled.

We present an application of two-stage sampling for estimating age composition in the Atlantic menhaden (*Brevoortia tyrannus*) purse-

seine fishery in this paper. The National Marine Fisheries Service, in its menhaden research program, has been collecting length, weight, and age data from landings made at reduction plants along the east coast of the United States. This information (along with records of plant landings, numbers of sets made, and estimates of catch per set by vessel captains) has been essential for determining age structure and seasonal migration (Nicholson 1971a, 1972; June 1972; Nelson et al. 1977), for evaluating fishing impact (Nicholson 1971b; Schaaf and Huntsman 1972; Schaaf 1979), and for developing management proposals (Schaaf 1975; Schaaf et al. 1975). One such proposal recommended by the Atlantic Menhaden Management Board in 1981 was that at least 10% of all fish landed be Age 3 or older. Nevertheless, few attempts have been made to calculate confidence intervals about estimated parameters such as the mean age composition of the catch.

In this paper, we (1) describe the current menhaden sampling procedure in terms of probability sampling theory, (2) discuss the assumptions of the underlying statistical model and consequences of their violation, (3) demonstrate methods for calculating approximate confidence intervals about age composition estimates, and (4) develop a graphical technique which yields an



**Figure 1. Ports at which Atlantic menhaden were landed during the 1979 fishing season.**

units. Both primary and secondary units are selected with equal probabilities and without replacement. A major assumption is that both sets and individual fish within sets are selected randomly.

Because only the last purse-seine set of a trip is actually sampled, the set is the primary sampling unit. We assume that sampling last sets yields a random sample from the population of all sets landed at a port during a given week. This assumption is satisfied if:

- (1) vessels randomly harvest schools of menhaden so that, in aggregate, last sets per trip are representative of all sets taken; and
- (2) sets are sampled at random from the total population of last sets landed at a port.

The first condition could be violated if age distribution is influenced by set location. For example, 35% of all vessels sampled in the Chesapeake Bay area during 1979 included sets taken both inside and outside the Bay. For these landings, a log-likelihood ratio test (*G* statistic) (Sokal and Rohlf 1981) indicated that last sets were more likely ( $P < 0.05$ ) to be made inside the Bay. Because average fish weight in sampled sets was greater outside (169.9 g) than inside (142.2 g) the Bay (Student's *t*-test,  $P < 0.05$ ), we concluded that the observed oversampling of inside sets may have introduced a 5.4% underestimate of average weight for these landings (Chester 1984). Average age presumably would be underestimated also.

The second condition would be violated if vessels were not selected randomly. Port samplers have been instructed to avoid following patterns or showing favoritism in vessel selection, but we found evidence that vessels were not sampled randomly in Chesapeake Bay during the 1979 fishing season (*G* statistic,  $P < 0.05$ ).

For each randomly selected set, the port sampler presently collects a bucket of fish from the top of the hold and randomly draws a 10-fish subsample. A good critique of the problem of randomly selecting fish from a vessel is given by Tomlinson (1971), who concluded that the task is operationally impossible. Nevertheless, we assume that any added bias is small.

#### ESTIMATING AGE COMPOSITION OF THE CATCH

Following Cochran (1977), the age composition of the catch for any particular port-week is

unambiguous solution to the problem of optimal allocation of sampling effort when several parameters (e.g., age proportions) are estimated simultaneously.

#### STRUCTURE OF THE SAMPLING DESIGN

In the present menhaden survey, sample data are reported by week for each active port (Fig. 1). Port sampling personnel are asked to randomly select vessels and obtain 10 fish from the top of each load. Thus, a sample is assumed to represent fish from the last set made, rather than from the entire boatload. For any given port-week combination, this sampling design may be classified as two-stage cluster sampling or subsampling (Cochran 1977), where purse-seine sets are the primary units of unequal population size and individual fish are the secondary sampling

estimated by averaging the relative proportions of each age class over  $n$  sampled sets:

$$\bar{p}_j = \frac{\sum_{i=1}^n p_{ij}}{n} \quad (1)$$

where  $p_{ij}$  denotes the proportion of fish in the  $j$ th age category for the subsample from the  $i$ th set. We assume that the age of each sampled fish, determined from scale readings, is known without error.

The estimator,  $\bar{p}_j$ , is subject to an inherent bias because sets are composed of unequal numbers of fish (Pope 1956; Cochran 1977). The bias arises because the inequality of set size affects the probability of including a given fish in a subsample. The bias may not be appreciable, however, if set sizes do not vary greatly, or if there is no correlation between set size and age composition and the number of sets sampled is large (Sukhatme and Sukhatme 1970). The bias potential is small for most of the Atlantic menhaden fishery because age composition does not vary drastically over the period of a week.<sup>1</sup> We assume equal set size in our subsequent discussion.

Cochran (1977) defined the sample variance as:

$$v(\bar{p}_j) = \frac{(1 - f_1)s_1^2}{n} + \frac{f_1(1 - f_2)s_2^2}{nm} \quad (2)$$

where

<sup>1</sup> The exception is a well-developed autumn fishery off Beaufort, North Carolina, where age composition varies greatly among sets, and where there is evidence that fish in larger sets are younger than those in smaller sets (Chester 1984). In this circumstance, the estimate of average age is subject to an upward bias. When equal set size cannot be assumed, a ratio-to-size estimator can be calculated by weighting according to set size:

$$\bar{p}_j = \frac{\sum_{i=1}^n M_i p_{ij}}{\sum_{i=1}^n M_i}$$

where  $M_i$  is an estimate of the number of fish in the  $i$ th set and found by dividing the estimate of total set weight (from the Captain's logbook records) by average weight per fish (from dockside samples).

$$s_1^2 = \frac{\sum_{i=1}^n (p_{ij} - \bar{p}_j)^2}{(n - 1)} \quad (3)$$

and

$$s_2^2 = \frac{m}{n(m - 1)} \sum_{i=1}^n p_{ij}(1 - p_{ij}) \quad (4)$$

Thus, there are two parts to the variance expression: variance among sampled sets,  $s_1^2$ , and variance within sets,  $s_2^2$  (the expected variance of a binomial proportion). The  $f_1$  and  $f_2$  terms are the fractions sampled of sets and of fish within sets, respectively ( $f_1 = n/N$ , where  $N$  is the number of sets landed;  $f_2 = m/M$ , where  $M$  is the number of fish per set). As  $f_1$  and  $f_2$  increase, overall variance decreases because progressively more information is known about the population. In the menhaden survey,  $f_2$  is always negligible and  $f_1$  is generally  $<0.10$ . For these reasons, the variance formula may be simplified:

$$v(\bar{p}_j) \approx \frac{s_1^2}{n} = \frac{\sum_{i=1}^n (p_{ij} - \bar{p}_j)^2}{n(n - 1)} \quad (5)$$

Equations 2 and 5 differ in form from that of Schweigert and Sibert (1983; equations 4a and 4b for a three-stage design) in that those authors selected Cochran's (1977) variance equation 10.34 which assumes that population variances (e.g.,  $S_1^2$  and  $S_2^2$ ) are known. We prefer Cochran's equation 10.36 which uses sample estimates ( $s_1^2$  and  $s_2^2$ ) (Smith 1984).

Because  $J$  age proportions are jointly estimated from the sample data, we recommend calculation of simultaneous confidence intervals (Goodman 1965; Tortora 1978), rather than confidence intervals estimated for each age individually (Cochran 1977). Confidence intervals for each age class are estimated in the traditional way,  $\bar{p}_j \pm t\sqrt{v(\bar{p}_j)}$ , except that  $t$  is chosen at a significance level of  $\alpha/J$  instead of  $\alpha$ . This choice of  $t$  guarantees an overall error rate of  $\alpha$ , and implies a probability of at least  $1 - \alpha$  that all  $J$  confidence intervals will include the corresponding true values.

For example, consider samples collected at Reedville, Virginia during the week ending September 1, 1979. The proportions of Age-2 fish for eight sampled sets were 0.9, 0.7, 0.5, 0.8, 0.8,

**Table 1.** Total catch of Atlantic menhaden per week (stratum), weighting factor per week, and mean age proportion ( $\bar{p}_j$ ) and variance ( $v(\bar{p}_j)$ ) for ages  $j = 0, 1, 2$ , and  $3+$  by week for Reedville, Virginia during the 1979 fishing season.

Week	Total catch ( $\times 10^6$ )	Weighting factor	Age 0		Age 1		Age 2		Age 3+	
			$\bar{p}_0(\%)$	$v(\bar{p}_0)$	$\bar{p}_1(\%)$	$v(\bar{p}_1)$	$\bar{p}_2(\%)$	$v(\bar{p}_2)$	$\bar{p}_3(\%)$	$v(\bar{p}_3)$
5/26	36.80	0.02646			2.5	$2.68 \times 10^{-4}$	96.3	$6.92 \times 10^{-4}$	1.3	$1.56 \times 10^{-4}$
6/2	55.30	0.03976					98.0	$1.78 \times 10^{-4}$	2.0	$1.78 \times 10^{-4}$
6/9	55.36	0.03980					97.0	$4.56 \times 10^{-4}$	3.0	$4.56 \times 10^{-4}$
6/16	69.01	0.04961			3.0	$4.56 \times 10^{-4}$	97.0	$4.56 \times 10^{-4}$		
6/23	59.66	0.04289			2.0	$1.78 \times 10^{-4}$	97.0	$2.33 \times 10^{-4}$	1.0	$1.00 \times 10^{-4}$
6/30	59.36	0.04267			23.0	$1.16 \times 10^{-2}$	77.0	$1.16 \times 10^{-2}$		
7/7	62.74	0.04511			22.9	$7.00 \times 10^{-3}$	77.1	$7.00 \times 10^{-3}$		
7/14	59.87	0.04304			18.0	$4.40 \times 10^{-3}$	82.0	$4.40 \times 10^{-3}$		
7/21	42.28	0.03040			18.2	$5.77 \times 10^{-3}$	81.8	$5.77 \times 10^{-3}$		
7/28	71.71	0.05155			8.0	$1.07 \times 10^{-3}$	92.0	$1.07 \times 10^{-3}$		
8/4	47.42	0.03409			9.2	$1.61 \times 10^{-3}$	90.8	$1.61 \times 10^{-3}$		
8/11	39.18	0.02817			11.2	$3.01 \times 10^{-3}$	79.4	$3.92 \times 10^{-3}$	9.3	$3.90 \times 10^{-3}$
8/18	67.21	0.04832			12.1	$1.73 \times 10^{-3}$	86.9	$2.23 \times 10^{-3}$	1.0	$1.00 \times 10^{-4}$
8/25	30.19	0.02170			7.0	$9.00 \times 10^{-4}$	88.0	$1.07 \times 10^{-3}$	5.0	$5.00 \times 10^{-4}$
9/1	44.76	0.03218			12.5	$3.12 \times 10^{-3}$	75.0	$2.50 \times 10^{-3}$	12.5	$3.84 \times 10^{-3}$
9/8	48.73	0.03503			47.2	$6.10 \times 10^{-3}$	52.8	$6.10 \times 10^{-3}$		
9/15	51.22	0.03682			65.0	$3.57 \times 10^{-3}$	35.0	$3.57 \times 10^{-3}$		
9/22	58.02	0.04171	2.0	$4.00 \times 10^{-4}$	32.1	$1.03 \times 10^{-3}$	58.7	$8.81 \times 10^{-3}$	7.2	$1.86 \times 10^{-3}$
9/29	48.03	0.03453	5.0	$1.17 \times 10^{-3}$	66.7	$4.44 \times 10^{-3}$	28.3	$4.94 \times 10^{-3}$		
10/6	48.14	0.03461			38.4	$8.21 \times 10^{-3}$	60.6	$8.12 \times 10^{-3}$	1.0	$1.00 \times 10^{-4}$
10/13	59.09	0.04248	1.2	$1.56 \times 10^{-4}$	65.0	$1.46 \times 10^{-2}$	33.8	$1.46 \times 10^{-2}$		
10/20	147.84	0.10628			78.7	$6.84 \times 10^{-3}$	21.3	$6.84 \times 10^{-3}$		
10/27	51.43	0.03697			68.6	$1.70 \times 10^{-2}$	28.6	$1.32 \times 10^{-2}$	2.7	$7.72 \times 10^{-4}$
11/3	45.16	0.03247			57.1	$1.57 \times 10^{-2}$	42.9	$1.57 \times 10^{-2}$		
11/10	32.47	0.02334	2.5	$2.68 \times 10^{-4}$	82.5	$1.53 \times 10^{-2}$	11.2	$7.66 \times 10^{-3}$	3.8	$1.41 \times 10^{-3}$

0.9, 0.8, 0.6. The mean proportion of Age-2 fish was estimated as 0.75, with a variance of 0.0025 (equations 1 and 5). To calculate a simultaneous 95% confidence interval, choose  $t$  at 0.05/ $J$ . Here, three age categories were found and  $t$  was chosen at the 0.017 significance level. The approximate confidence interval was  $0.75 \pm 0.12$ . Estimates of mean age proportions and their variances are displayed for Reedville for each week during the 1979 fishing season (Table 1).

In practice, the average age composition must be known for time intervals longer than 1 week. To estimate age composition at a single port for the entire season, we assume a stratified two-stage design in which  $W$  individual weeks are strata whose relative population sizes are indexed by estimates of total number of fish landed per week. For a given port over the entire season:

$$\bar{p}_{js} = \frac{\sum_{w=1}^W N_w \bar{p}_{jw}}{\sum_{w=1}^W N_w} = \sum_{w=1}^W b_w \bar{p}_{jw} \tag{6}$$

where  $\bar{p}_{js}$  is the overall average proportion of the  $j$ th age category for the season,  $N_w$  is the estimated catch (in numbers) for the  $w$ th week, and  $\bar{p}_{jw}$  is the average proportion of age  $j$  fish by week. Therefore,  $b_w$  is a week-specific weighting factor. The variance of this overall seasonal proportion is:

$$v(\bar{p}_{js}) = \sum_{w=1}^W b_w^2 v(\bar{p}_{jw}) \tag{7}$$

assuming that the  $\bar{p}_{js}$  are independent among weeks. Age composition estimates and approximate simultaneous 95% confidence intervals are given for Reedville for the 1979 fishing season (Table 2).

OPTIMAL ALLOCATION OF  
SAMPLING EFFORT

When designing surveys, scientists need to decide how limited resources (time or money) are to be allocated among sampling stages. In the two-stage design for menhaden, this amounts to determining the optimal number of sets ( $n$ ) and

**Table 2.** The overall seasonal age proportions ( $\bar{p}_j$ ), variances, and simultaneous 95% confidence intervals for Atlantic menhaden landed at Reedville, Virginia during 1979.

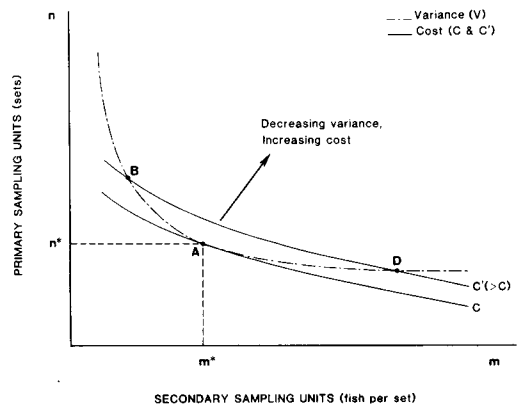
Age	$\bar{p}_j$ (%)	$v(\bar{p}_j)$	Confidence intervals
0	0.37	$2.51 \times 10^{-6}$	$\pm 0.40$
1	32.28	$2.62 \times 10^{-4}$	$\pm 4.05$
2	65.73	$2.54 \times 10^{-4}$	$\pm 3.98$
3+	1.62	$1.40 \times 10^{-5}$	$\pm 0.94$

fish within sets ( $m$ ) to be sampled. Procedures for determining optimal two-stage designs for a single parameter are well-documented (Cochran 1977; Sokal and Rohlf 1981). However, determining the optimal design for age composition involves several simultaneously measured parameters. The multiple parameter case has been treated for benthic invertebrate species (Saila et al. 1976) and for age classes of fish (Schweigert and Sibert 1983). Both studies treated each parameter separately so that investigators were left to choose, on an ad hoc basis, the eventual sampling design from among the individual optima. An analytical solution to the problem involves minimizing a nonlinear cost function subject to several variance constraints (Kelley 1971). Such a procedure is computationally formidable, and we believe a graphical solution to be more practical.

We begin our graphical solution by developing optimal designs for individual age categories. The variance of each mean age proportion ( $\bar{p}_j$ ) depends on the among set and within set variance components,  $s_a^2$  and  $s_w^2$  (Table 3):

$$v(\bar{p}_j) \approx \frac{s_1^2}{n} = \frac{s_a^2}{n} + \frac{s_w^2}{nm} \quad (8)$$

Note that different combinations of  $n$  and  $m$  yield the same variance (line  $V$  in Fig. 2). Increases in



**Figure 2.** Least-cost sampling strategy for a given variance,  $V$ , in a two-stage sampling design. The minimum cost of achieving variance  $V$  occurs with  $n^*$  primary and  $m^*$  secondary sampling units (Point A). Points B and D also satisfy the variance constraint but at greater cost.

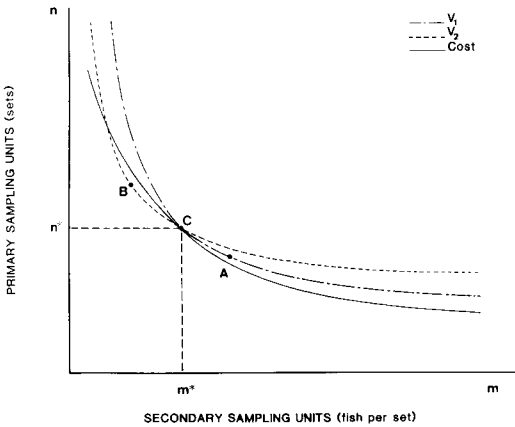
both  $n$  and  $m$  reduce variance; therefore, successively higher curves represent greater precision (i.e., lower variance), and successively lower curves represent reduced precision (i.e., greater variance). Graphically, the desired precision (i.e., the variance constraint) of the mean age proportion is an iso-variance line whose location is determined by (1) the maximum acceptable difference between the estimated ( $\bar{p}_j$ ) and true ( $\mu_j$ ) means, and (2) the level of confidence with which that difference is to be attained. Relaxing the variance constraint shifts the iso-variance curve towards the origin.

The optimal  $n$ - $m$  combination yields the desired precision at the lowest cost. Sampling cost ( $C$ ) is assumed to be proportional to the number of sets and number of fish sampled:

$$C = c_1n + c_2nm \quad (9)$$

**Table 3.** Analysis of variance of a two-stage sampling design for a single age proportion.

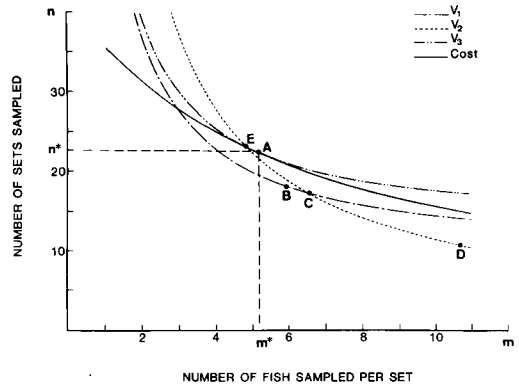
Source	df	Mean square	Variance components
Among primary units	$n - 1$	$ms_1^2 = m \sum_{i=1}^n \frac{(p_{ij} - \bar{p}_j)^2}{(n - 1)}$	$s_w^2 + ms_a^2$
Within primary units	$n(m - 1)$	$s_2^2 = \frac{m}{n(m - 1)} \sum_{i=1}^n p_{ij}(1 - p_{ij})$	$s_w^2$



**Figure 3.** Optimal allocation of sampling effort in a two-stage design when two parameters are simultaneously estimated. The solution set consists of the optimum for each constraint considered individually (points A and B) and the intersection of both constraints (Point C). In this example, only Point C satisfies both variance constraints.

where  $c_1$  is the cost of sampling a set, and  $c_2$  is the cost of aging an individual fish. The optimal  $n$ - $m$  combination for  $\bar{p}_i$  is determined at the point of tangency between the cost curve and the variance constraint (Fig. 2).

The traditional approach is to solve for optimal  $n$  and  $m$  for each age class, then choose the final survey design from among the individual optima (e.g., Schweigert and Sibert 1983). The desired optimal solution is taken as the least costly  $n$ - $m$  combination that satisfies all variance constraints but, when several parameters are measured simultaneously, it is possible that no individual solution can satisfy all constraints. Consider an example with two variance constraints where neither of the individual optima satisfies both constraints (Fig. 3). Point A minimizes the sampling cost of satisfying Constraint 1 but does not satisfy Constraint 2. Similarly, Point B minimizes the cost of satisfying Constraint 2 but fails to simultaneously satisfy Constraint 1. The intersection of constraints (Point C) demands a higher sampling budget than either A or B, but it is the least expensive way of simultaneously satisfying both variance requirements. Thus, the complete solution set for op-



**Figure 4.** An example of the optimal two-stage sampling strategy for estimating mean proportions of age 1, 2, and 3+ fish in the Atlantic menhaden fishery. Curve  $V_i$  denotes the variance constraint for Age- $i$  fish. Points A-E represent candidates for a solution, with  $n^*$ - $m^*$  (Point A) indicating the optimal number of sets and fish per set to be sampled. Data pertain to the week ending September 1, 1979 at Reedville, Virginia.

timal allocation in a two-stage design includes the intersections (if they exist) of each pair of variance constraints as well as the optimum for each age category considered individually.

We illustrate the calculation of optimal survey design for the Reedville example introduced earlier. During the week ending September 1, 1979, approximately 12.5% of the menhaden catch were Age-1 fish, 75% were Age 2, and the remaining 12.5% were Age 3. The among set and within set variance components and the desired precision determine the shape and location of each variance constraint (Fig. 4). We require that each mean proportion be estimated with an accuracy of  $\bar{p}_i - \mu_i = 0.1$ , with a joint confidence level for all three parameters of 95%. Given a cost ratio of  $c_1:c_2 = 6$ , the least expensive way to satisfy all three constraints is to sample  $n = 23$  sets and  $m = 5$  fish per set (Point A in Fig. 4). Other candidates for the solution include points B-E, but they either fail to simultaneously satisfy all three constraints (e.g., points B, C, and D), or they represent a more costly way of satisfying all constraints (e.g., Intersection E). Finally, we emphasize that this example is only to illustrate the principles of determining optimal survey design

and does not represent the optimal allocation of sampling effort for the entire Atlantic menhaden fishery.

### DISCUSSION

Incorporation of probability sampling theory into the design of fishery surveys allows confidence statements to be made about stock assessment conclusions but, because of practical problems of sampling commercial catches, the application of these theories rarely is straightforward. The assumption of random sample selection is particularly critical to ensure unbiased parameter estimates.

The bias caused by violating the random sampling assumption depends on the nature of the fishery. For Atlantic menhaden, the potential bias varies with sampling stage, port, and time of year. In Chesapeake Bay, where the population of sets available for sampling (i.e., last sets) may not be representative of all sets landed, the bias should be estimated and accounted for. We also recommend more formal randomization procedures for sampling vessels. Bias incurred by relaxing the assumption of random sample selection also depends on the variability among sets landed at a particular port. North of Chesapeake Bay, the variability among sets is small within each week and nonrandom selection would introduce little bias. In the Chesapeake Bay area, an inside-outside variation in average age and a tendency to oversample inside sets may combine to cause slight underestimates of the proportion of older fish. At Beaufort, North Carolina, an inverse relationship between numbers of fish per set and average age may cause a significant overestimate of older fish unless estimates are weighted by set size. The consequences of nonrandom selection also may be related seasonally. Seasonal recruitment and migration in the Atlantic menhaden fishery cause the variation among sets to increase through the season; hence, the potential for choosing a nonrepresentative sample also increases.

In addition to the practical aspects of sampling commercial catches, the survey design must provide for the specific data needs of stock assessment scientists. Our experience indicates that variances of age proportions estimated on a weekly basis are relatively large in the menhaden fishery (Table 1). Specifying rigorous variance constraints can result in an unattainable sam-

pling intensity, particularly at Beaufort where the optimal sample size may exceed the number of sampling opportunities. However, if scientists require only seasonal estimates, then current levels of sampling effort are probably sufficient—at least for estimating Age-1 and Age-2 Atlantic menhaden (Table 2).

Optimality theory offers the potential to improve survey design by accounting for seasonal and spatial patterns of variance in a fishery. For Atlantic menhaden, within set components of variance tend to be small. Therefore, survey effort should focus on the number of sets chosen and should be most intense at those places and times that among-set variance, is largest. The most efficient design, then, implies greatest effort at ports such as Beaufort where age composition is most variable. Moreover, sampling effort should vary seasonally if predictable trends in among set variance occur.

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